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OPTIMIZATION OF THE DYNAMIC PROPERTIES OF ADAPTIVE PRESSURIZED STRUCTURES SUBJECTED TO IMPACT LOADS.

Cezary Graczykowski*, Jan Holnicki-Szulc*

* Institute of Fundamental Technological Research, SMART- TECH Centre
Polish Academy of Sciences
Świętokrzyska 21, 00-049 Warsaw, Poland
e-mail: cgraczyk@ippt.gov.pl, holnicki@ippt.gov.pl
web page: http://smart.ippt.gov.pl

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Abstract. *The paper presents briefly the concept of controlling dynamic response of the thin-walled structures by filling them with compressed air and its controlled release during the impact process. The succesful result of using this method is strongly dependent on a proper pressure adjustment inside the structure according to the impact type and direction. The paper is focused on developing strategy for optimal distribution and release of pressure and corresponding software tools. This software contains procedures for numerical simulation of the impact process and optimization algorithms. Considered objective functions are formulated basing on conditions concerning: possibly soft absorption of the impact, dissipating the highest amount of energy, using the lowest pressure values, achieving assumed deformation of the structure. Various methods for solving derived optimization problems are proposed. Solutions are obtained for several impact scenarios and structure geometries. The results prove that pressure adjustment strategy has a significant influence on the deformation process and on energy absorption. Conducted simulations indicate that pressurised structure can easily change its dynamic properties and is possible to adapt to variable load cases and impact types.*

1 INTRODUCTION

Thin-walled structures are commonly used in car and mechanical industry because of their huge durability, stiffness and small weight. They efficiently absorb energy of the front impact due to the process of folding. Estimation of critical dynamic forces and dissipation capabilities for several types of thin-walled structures obtained by simplified analytical models can be found in classical crashworthiness literature cf. Johnes[(1)]. The problem of increasing the load capacity of such structures under axial loading by using pressure, structural fuses and pyrotechnik detachable connectors was successfully examined by Gren[(2)], Knap[(3)], Ostrowski, Griskevicius and Holnicki-Szulc[(4)].

In this paper we focus our attention on lateral impact where substantial difficulties might appear. The thin-walled structure undergoes large deformations and local plastic yielding but only small part of the impact energy can be dissipated. Significant improvement of the structure properties by filling it with compressed air and its controlled release was proposed by authors in their previous paper on this topic cf. Graczykowski, Chmielewski, Holnicki-Szulc[(5)]. Filling with gas in the initial stage of the lateral impact results in increasing the load capacity of the structure and prevents its huge deformations. The purpose of the controlled release of pressure is to dissipate energy accumulated in gas and to confine accelerations to admissible level. Initial value and change of pressure in time must be adjusted according to the type and direction of impact. Additional advantages can be achieved by dividing structure into pressurized packages and controlling values of pressure in every cell separately. Compressed air has also a beneficial influence on the buckling behavior of the structure, since it increases the value of critical force. These features of the pressurized structures were confirmed by experiment conducted on aluminium can. Structures adapted to impact by using compressed gas will be further called Adaptive Pressurized Structures (APS).

2 CORRESPONDING OPTIMIZATION PROBLEMS

This paper is aimed at developing a strategy for optimal distribution and release of gas in pressurized packages. For this purpose the above considerations are formulated as optimization problem. The structure being optimized is a simple two dimensional frame (cf. Fig.1) which could serve as a basis for a car door design. The structure may be divided into various number of packages and it may have clamped or sliding supports. The frame is loaded on the upper beam which is modeling lateral (non-axial) impact. Material and geometrical nonlinearities are taken into account. The material is elasto-plastic with the hardening, large deformations of the frame are considered.

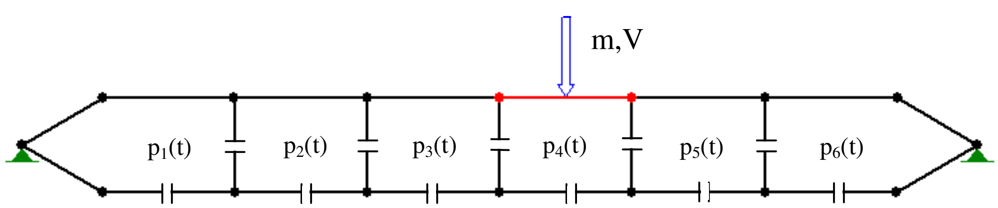

Figure 1: Structure divided into pressurized packages considered in the optimization problem.

The optimization is performed with respect to changes of pressure in every package. The objective function is based on the following criteria: dissipation of the highest impact energy, possibly soft absorption of impact, applying the smallest pressure values for given impact, achieving assumed deformation of the structure. The subsequent optimization problems, together with methods of solution and results are presented in the further part of the paper. Derived objective functions and constraints could have a fairly complicated implicit form since they are obtained from nonlinear dynamic analysis. Constraints are formulated basing on kinematic conditions imposed on maximal displacement and on the shape of frame destruction.

Solving mentioned problems requires using Finite Element software and optimization procedures. In this case a linkage between Ansys and Matlab packages is effectively used. The major application is Matlab, in which model parameters are stored and a batch file for FEM analysis is created. Ansys is launched within Matlab to conduct nonlinear transient dynamic analysis by means of Newmark algorithm and Newton-Raphson procedure, cf.[6] and to calculate value of the objective function. Afterwards this function is minimized by Matlab built-in optimization procedures.

3 DISSIPATION OF THE HIGHEST ENERGY

3.1 Mathematical formulation

This chapter concerns examination of overall properties of the pressurized structure. On this stage we do not assume a detailed impact scenario, but we investigate how much we can improve the structure by pressurizing. Hence the objective function of the optimization problem will be based on condition of dissipating the highest impact energy.

The dynamic system is described by well-known equation of motion in its nonlinear form with initial conditions imposed on unknown function and its first derivative:

$$M\ddot{q}(t) + C\dot{q}(t) + K(q)q(t) = F(q,t) \tag{1}$$

$$q(0) = q_0 \ , \ \dot{q}(0) = \dot{q}_0$$

The impact subjected to the structure upper beam is modeled by concentrated mass of the hitting object which is included in mass matrix and its velocity being one of the initial conditions. We assume that the impact velocity is established so the load capacity will be measured only by the hitting object mass. Values of pressures inside the packages influence right hand side load vector F and displacements q likewise. Thus, dynamic properties of the structure can be controlled by means of internal pressure. The solution to the problem (1) can be written by implicit nonlinear vector function $\mathbf{q}(m,\mathbf{p},t)$ where m is the mass of the hitting object, vector $\mathbf{p} = \{p_1(t), p_2(t),..., p_n(t)\}$ contains functions describing change of pressure in every cell. Function $q_m(m,\mathbf{p},t)$ is a component of $\mathbf{q}(m,\mathbf{p},t)$ and denotes displacements of the node to which impact is subjected. We will often use $\mathbf{q}(m,\mathbf{p},t_{stop})$ indicating deformation of the structure at the moment of braking the mass i.e. at time when velocity of the hitting object approaches zero for the first time. Arbitrary assumed set of all vectors of pressures will be denoted Ω_p and set of all values of mass

Ω_m. Both sets are arbitrary restricted to positive values. Additionally values of functions $p(t)$ being components of vector $\mathbf{p} \in \Omega_p$ can not exceed a fixed upper limit.

Let us introduce two scalar functions describing deformation of the structure. The function $D(m,\mathbf{p},t)$ measures distance between concentrated mass and the closest node of the lower span at certain moment in time:

$$D(m,\mathbf{p},t_x) = dist(q_m(m,\mathbf{p},t_x), q_l(m,\mathbf{p},t_x)) \tag{2}$$

The second function describes the largest penetration i.e. the largest vertical displacement of the lower beam which is expected to occur at the moment of braking the mass $q_{\max}(m,\mathbf{p},t_{stop})$. We will use kinematic approach, which means that the conditions defining destruction of the structure will be based on its deformation shape. The deformation of the structure is assumed to be admissible if there is no collision between the mass and the lower span before the moment of braking the mass and when maximal displacements of the lower span do not exceed limit value. We can define set S_q which contains admissible deformation of the frame as:

$$S_q = \{\mathbf{q}(m,\mathbf{p},t) \text{ satisfying conditions :} \tag{3}$$

$$1)\ D(m,\mathbf{p},t) > 0 \quad \text{for } t \in (0, t_{\text{stop}})$$

$$2)\ q_{\max}(m,\mathbf{p},t_{stop}) \le q_{adm} \ \}$$

Engineering formulation of the optimization problem is as follows: find maximal mass which can be subjected to the structure appropriately filled with pressure and do not violate kinematic conditions imposed on structure deformation. Thus we are searching for maximal mass and corresponding distribution of pressure as well. Straightforward mathematical formulation can be written:

$$\text{Find: } \max_{m,\mathbf{p}} \{ m \mid \mathbf{q}(m,\mathbf{p},t) \in S_q \} \tag{4}$$

where S_q is defined by (3)

The initial set of design variables $\Omega_m \times \Omega_p$ is here strongly restricted by conditions (3) imposed on deformation. In many cases the above optimization task can not be solved efficiently. Let us simplify the problem (4) and rearrange it to a form more convenient for numerical calculations. We can define function $S_m(\mathbf{p})$ returning subset of Ω_m which contains values of mass causing admissible deformation after impact on the structure with assumed change of pressures in all chambers:

$$S_m(\mathbf{p}) \subseteq \Omega_p \tag{5}$$

$$S_m(\mathbf{p}) = \{ m \in [0, m_{\max}] \mid \mathbf{q}(m,\mathbf{p},t) \in S_q \}$$

The scalar function $\Phi_m(\mathbf{p})$ will indicate load capacity of the structure i.e. maximal mass which can be subjected to the frame filled with given pressures.

$$\Phi_m(\mathbf{p}) = \max\{m \in S_m(\mathbf{p})\} = \max\{m \in [0, m_{\max}] \mid \mathbf{q}(m,\mathbf{p},t) \in S_q\} \tag{6}$$

Using function $\Phi_m(\mathbf{p})$ the problem (4) can be decomposed into the form:

$$\text{Find: } \max\{\Phi_m(\mathbf{p})\},\ \mathbf{p} \in \Omega_p \tag{7}$$

$$\text{where } \Phi_m(\mathbf{p}) = \max\{m \in [0, m_{\max}] \mid \mathbf{q}(m,\mathbf{p},t) \in S_q\}$$

and S_q is defined by (3)

We can notice that formulation (7) contains of two separate stages. First, we have to find maximal mass causing admissible deformation and then maximum of this function over pressure.

We will define limiting admissible deformations of the frame. A condition of distance between the spans being strictly positive is still satisfied when this distance approaches its minimum equal to zero exactly at the moment of braking the mass. Hence, we can write:

$$\Gamma^1_{Sq} = \{\ \mathbf{q}(m,\mathbf{p},t_{stop}) \text{ satisfying conditions:} \tag{8}$$

$$1)\ D(m,\mathbf{p},t_{stop}) = 0$$

$$2)\ \left.\frac{dD(m,\mathbf{p},t)}{dt}\right|_{t=t_{stop}} = 0$$

$$3)\ \left.\frac{d^2D(m,\mathbf{p},t)}{dt^2}\right|_{t=t_{stop}} > 0\ \}$$

This approach lets us investigate only the time at vicinity of t_{stop} and not the whole time of the transient analysis and simplify corresponding formulas. The limit value of second condition introduced in (3) can be obtained by rewriting it in the form of equality:

$$\Gamma^2_{Sq} = \{\ \mathbf{q}(m,\mathbf{p},t_{stop}) \text{ satisfying condition:} \tag{9}$$

$$q_{\max}(m,\mathbf{p},t_{stop}) = q_{adm}\ \}$$

By analogy to (5) we can define a scalar function of pressure $\Gamma^1_{Sm}(\mathbf{p}) = m^1(\mathbf{p})$ which returns one-element subset of $S_m(\mathbf{p})$ containing mass causing contact of the spans at the moment of braking the mass:

$$\Gamma^1_{Sm}(\mathbf{p}) \subseteq S_m(\mathbf{p}) \tag{10}$$

$$\Gamma^1_{Sm}(\mathbf{p}) = \left\{ m \in [0, m_{\max}] \,|\, \mathbf{q}(m, \mathbf{p}, t_{stop}) \in \Gamma^1_{Sq} \right\}$$

and scalar function of pressure $\Gamma^2_{Sm}(\mathbf{p}) = m^2(\mathbf{p})$ which returns a one-element subset of $S_m(\mathbf{p})$ containing mass causing limit value of lower span displacement at the moment of braking the mass:

$$\Gamma^2_{Sm}(\mathbf{p}) \subseteq S_m(\mathbf{p}) \tag{11}$$

$$\Gamma^2_{Sm}(\mathbf{p}) = \left\{ m \in [0, m_{\max}] \,|\, \mathbf{q}(m, \mathbf{p}, t_{stop}) \in \Gamma^2_{Sq} \right\}$$

To find function $\Phi_m(\mathbf{p})$ we have to choose a smaller of the functions $\Gamma^1_{Sm}(\mathbf{p})$ and $\Gamma^2_{Sm}(\mathbf{p})$:

$$\Phi_m(\mathbf{p}) = \begin{cases} \Gamma^1_{Sm}(\mathbf{p}) & when \quad \Gamma^1_{Sm}(\mathbf{p}) \le \Gamma^2_{Sm}(\mathbf{p}) \quad \text{(a)} \\ \Gamma^2_{Sm}(\mathbf{p}) & when \quad \Gamma^1_{Sm}(\mathbf{p}) > \Gamma^2_{Sm}(\mathbf{p}) \quad \text{(b)} \end{cases} \tag{12}$$

Taking into account the condition (12) we can divide the set of pressures Ω_p into two subsets Ω^1_p and Ω^2_p where (12a) and (12b) are fulfilled, respectively.

The second step in solving the optimization problem (7) is finding maximum of $\Phi_m(\mathbf{p})$. Using for this purpose gradient based methods is not effective since function $\Phi_m(\mathbf{p})$ is not smooth as composed of functions Γ^1_{Sm} and Γ^2_{Sm} . The maximum $\Phi_m(\mathbf{p})$ which indicates maximal mass is expected to be achieved when both limit conditions imposed on deformation are fulfilled. This assumption is based on the fact that for insufficient pressure load capacity is exhausted by collision of the beams while for excessive pressure by exceeding the limit displacements. Hence maximum mass can be found for pressure vector $\mathbf{p} \in \Omega^{12}_p \subset \Omega^1_p$ found as a solution of the equation:

$$\Gamma^1_{Sm}(\mathbf{p}) = \Gamma^2_{Sm}(\mathbf{p}) \tag{13}$$

Another method of finding subset Ω^{12}_p is calculating deflection of lower span caused by mass $\Gamma^1_{Sm}(\mathbf{p})$:

$$q^1_{\max}(\mathbf{p}) = q_{\max}(\Gamma^1_{Sm}(\mathbf{p}), \mathbf{p}, t_{stop}) \tag{14}$$

and comparing to the limit value.

$$q^1_{\max}(\mathbf{p}) = q_{adm} \tag{15}$$

Equation (15) is more convenient for numerical calculations than equation (13). Having $\Gamma^1_{Sm}(\mathbf{p})$ calculated $q^1_{\max}(\mathbf{p})$ is obtained directly from solution of (1) while calculating $\Gamma^2_{Sm}(\mathbf{p})$ requires solving an inverse problem. Finally, the optimization task assumes the form:

$$\text{Find: } \max\left\{\Gamma^1_{Sm}(\mathbf{p})\right\},\ \mathbf{p} \in \Omega^{12}_p \tag{16}$$

where Ω_p^{12} is defined by (15)

Values of mass corresponding to each element of Ω_p^{12} are causing deformation with contact of the spans and limit displacement of the lower beam at the moment of braking the mass. We manage to restrict the initial set of optimization variables $\Omega_m \times \Omega_p$ to the set Ω_p^{12} containing vectors of pressure appropriately preselected to give maximal value of the corresponding mass.

3.2 Numerical example

Derived mathematical formulation can be applied for finding optimal distribution of pressures in three-cell pressurized structure fixed with no sliding (cf. Fig. 3). The loading is performed by object hitting the frame in the middle of the upper span with initial velocity equal to 2m/s. We assume that pressures inside packages do not exceed 1600 kN/m and that they are constant during the whole analysis. Impact subjected to this structure causes large deformations resulting in change of packages capacity which must be taken into account. To keep the pressure on constant level we have to release the volume of air equal to change of chamber capacity e.g. by opening exit valves. Vector **p** has two constant components p_1 and p_2 which indicate pressures in lateral and middle cell, respectively. Hence, introduced functions of pressure may be illustrated as corresponding surfaces, see Fig. 2a. Initially, we have to calculate the function (surface) of mass causing collision of the beams $m^1(\mathbf{p})$, which for low pressures is smaller than mass causing limit displacement $m^2(\mathbf{p})$. This procedure is expensive numerically because it requires multiple solving of inverse nonlinear dynamic problem. A surprising linearity of this surface will be effectively used later on. In the second step we find the corresponding function of maximal lower span displacements $q^1_{\max}(\mathbf{p})$. Intersection of this surface with the constant value of limit displacement (equal here 0,18m) constitutes line Ω_p^{12} which is the solution of equation (15) and (13) as well. This line is projected into surface $m^1(\mathbf{p})$ for the purpose of finding maximal mass, see Fig. 2a. Finally the load capacity is increased 6,1 times and is achieved for the maximal pressure p_1, cf. Table 1.

$\mathbf{p}_1(0)$ [kN/m]	$\mathbf{p}_2(0)$ [kN/m]	$\mathbf{p}_1(t_{stop})$ [kN/m]	$\mathbf{p}_2(t_{stop})$ [kN/m]	$q_1^{\max}$ [m]	t_{stop} [s]	**m** [kg]
0	0	0	0	0,04	0,152	**7596**
1600	1147	1600	1147	0,18	0,213	**46374**
400	3925	0	0	0,18	0,265	**68489**

Table 1. Comparison of structure load capacity for different values of pressure (central impact)

The second case considered concerns linear decrease of pressure in all packages. The time when pressure approaches zero is assumed to be equal to the time of braking the mass $m^1(\mathbf{p})$. This time is calculated beforehand and changes within the range 0,15 - 0,3s according to the initial values of p_1 and p_2, which are limited to 4800 kN/m. Both surfaces in Fig. 2b are calculated in terms of vector of initial pressures $\mathbf{p_0} = \{p_1, p_2\}$. In this case, the function $m^1(\mathbf{p}_0)$ achieves higher values than previously. The shape of surface $q^1_{\max}(\overline{\mathbf{p}}_\mathbf{0})$ is more sensitive to the value of pressure p_1, which results in different shape of line Ω^{12}_p, see Fig 2b. The highest mass is 9,01 times larger than the initial one and it is found for low value of pressure p_1. Detailed results for both cases are presented in Table 1 and corresponding deformation of the structure is presented in Fig. 3.

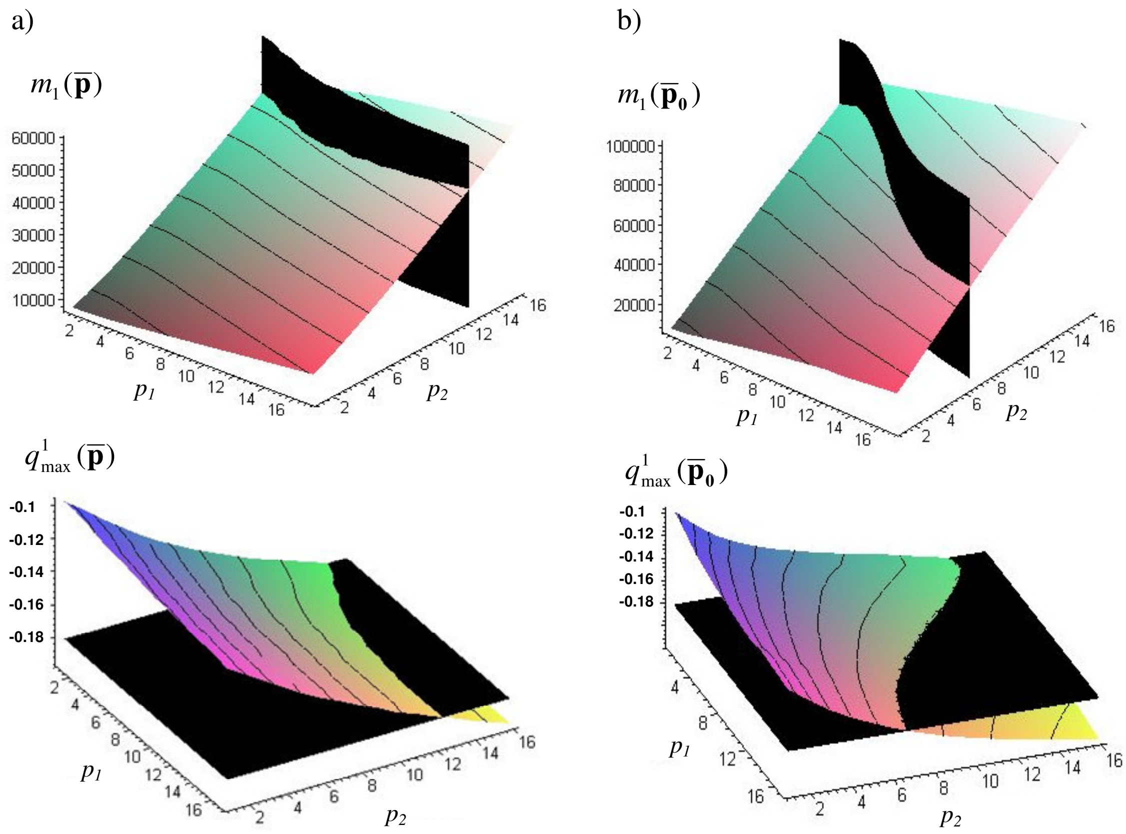

Figure 2: Surfaces $m^1(\overline{\mathbf{p}})$ and $q^1_{\max}(\overline{\mathbf{p}})$ in terms of non-dimensional pressure $\overline{\mathbf{p}}$, corresponding to the cases: a) constant value of pressure, $\overline{\mathbf{p}} = \mathbf{p}/(100\text{kN/m})$; b) linear decrease of pressure, $\overline{\mathbf{p}}_0 = \mathbf{p_0}/(300\text{kN/m})$.

Both problems described above were also solved without a huge numerical effort coming from the necessity of calculating unknown functions for every possible value of pressure. Functions finding mass $m^1(\mathbf{p})$ and corresponding displacement $q^1_{\max}(\mathbf{p})$ were defined in Ansys environment. Matlab built-in optimization procedure was used to find combinations of pressure for which function $q^1_{\max}(\mathbf{p})$ approaches limit value, what indicates line Ω^{12}_p. Another Matlab procedure was applied to find maximal mass on this line. Although used functions have a complicated form, time of calculations was much shorter than in the case of subsequent searching through pressure area.

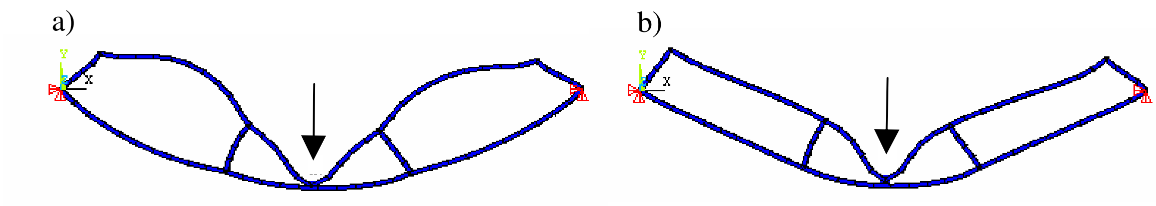

Figure 3: Deformation of the optimally pressurized structure loaded by maximal mass:
a) constant pressure; b) linear release of pressure

Load capacity can be also examined in the case of impact situated over the left chamber. Kinematic criteria for admissibility of structure deformation remain valid. Initial load capacity of the structure is over 40% smaller than in the case of central impact and it is exhausted when collision between upper and lower beam occurs. Pressurizing central package without release is not beneficial, since it causes excessively large outside displacements in central package and decrease of distance between spans in lateral chambers. Pressurizing only right package has almost no influence on dynamic properties of the frame. Due to these facts only filling left package with compressed air will be examined precisely. Displacement of the lower span is the largest below left package but it remains relatively small (0,121m) even for highest arbitrary assumed pressure value of 2500 kN/m and corresponding mass 52000 kg. Structure deformation is on the limit due to condition (8) as in the case without applying pressure.

$\mathbf{p}_1(0)$ [kN/m]	$\mathbf{p}_1(t_{stop})$ [kN/m]	$q_{\max}$ [m]	t_{stop} [s]	**m** [kg]
0	0	0,087	0,112	**4350**
2500(max)	2500(max)	0,121	0,192	**52000**
5000(max)	0	0,143	0,248	**76000**

Table 2. Comparison of structure load capacity for different values of pressure (lateral impact)

Similar situation occurs when linear decrease of pressure is assumed and time of pressure being zero is achieved at t_{stop}. The most beneficial is pressurizing only left package. After applying maximal arbitrary assumed pressure equal initially 5000kN/m mass of 76000 kg can be subjected to the structure. Load capacity is still exhausted due to collision of spans. Lower beam displacement is equal 0,143 m so it does not exceed limit value. When both left and central chamber are pressurized, load capacity is increased over 18 times in comparison to the initial one and is equal about 79000kg. The necessity of pressurizing two packages and, by doing so, introducing more energy to the system is a great disadvantage of this second solution.

In the two above investigated cases the optimal solution is not achieved on the line of intersection of two limit surfaces $m^1(\mathbf{p})$ and $m^2(\mathbf{p})$ because this line is situated outside the area of admissible pressures. In other words, the set of solutions of equation (15) Ω_p^{12} is not a subset of Ω_p. Calculating $m^1(\mathbf{p})$ for the highest value of pressure p_1 is sufficient to find maximal load capacity of the structure.

4 SOFT ABSORPTION OF THE IMPACT

The second purpose of applying pressure into the structure is to alleviate the impact by changing structure stiffness during collision. In this section we will assume given scheme of impact so mass of the hitting object will not be design variable in further analysis. Our goal is to control displacement, velocity and acceleration of the hitting object. The most expedient trajectory is a second order curve. By using this curve we can obtain linear descent of the velocity and constant value of acceleration during the whole process. Displacement of the node with applied mass is assumed to change according to the equation:

$$q_m^{opt}(t) = V_0 t + \tfrac{1}{2} a t^2 \tag{17}$$

where: initial velocity of the object $V_0 = 2m/s$, level of acceleration $a = -20m/s^2$ and time of the whole process $t = 0{,}1s$. Assumed trajectory (17) must be always situated above initial displacement curve since applying pressure we cannot increase compliance of the structure. The advantage of using this procedure is reduction of penetration by hitting object.

Mathematical formulation of the optimization problem can be written as follows:

$$\text{Find: } \min\left\{\Phi(\mathbf{p}(t)) \;\middle|\; \mathbf{q}(m,\mathbf{p},t) \in S_q\right\} \tag{18}$$

$$\text{where } \Phi(\mathbf{p}(t)) = \int_0^{t_{stop}} \left[q_m(t) - q_m^{opt}(t)\right]^2 dt$$

and $q_m(t)$ is the component of nonlinear implicit function $\mathbf{q}(m,\mathbf{p},t)$. The objective function is defined here as an integral obtained from a difference between actual and assumed displacement calculated over a given time domain. This formulation has a variational form since the argument of the minimized functional is a function of pressure in terms of time. Such formulated problem seems quite difficult to solve since we have to search through infinite number of functions $\mathbf{p}(t)$ and assuming this function as a polynomial does not give satisfactory results. However, while using time integration within Finite Element Method we can discretize the objective function in a given time domain as we do it with function of displacements. This way we obtain decomposition of the initial optimization problem into series of simpler ones, where objective function and design variables are defined in every moment in time:

$$\Phi(\mathbf{p}(t_k)) = \left[q_m(t_k) - q_m^{opt}(t_k)\right]^2 \tag{19}$$

The introduced optimization problem can be solved for several number of packages in the structure, however pressurizing only one of them is usually sufficient to fit both displacement curves. The most efficient is pressurizing the chamber to which impact is subjected. Structure here considered consists of only one package and has sliding support. A mass hitting the structure is equal to 500kg. Pressure in every single time step is adjusted to fit the displacement of the mass to the assumed curve. Results obtained from the previous steps are effectively used on the following ones. Ansys built-in procedures are applied to conduct optimization process. The resulting change of pressure is shown in Fig 4a. High value of pressure is necessary at the beginning of the impact and then the curve of pressure is declining gradually.

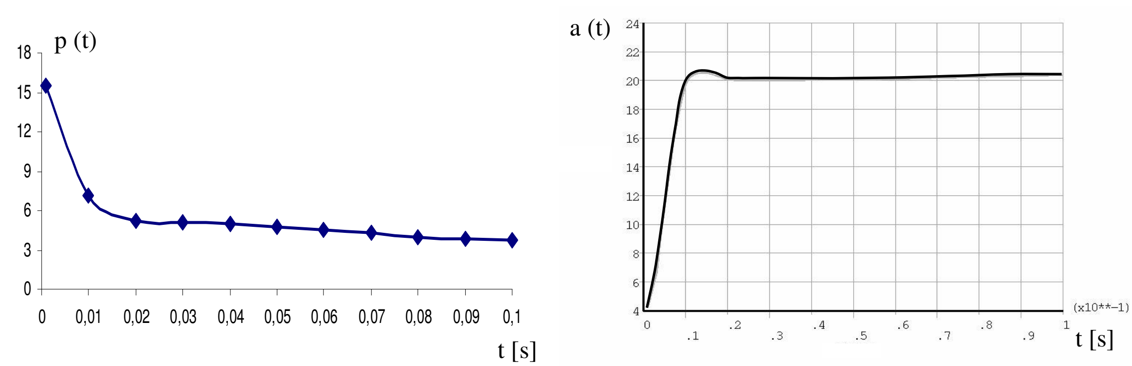

Figure 4: Results of the optimization: a) change of pressure in time; b) resulting acceleration of the hitted joint

In this example the objective function was diminished almost to zero so the assumed and obtained curves overlap and the level of the acceleration is almost constant, see Fig. 4b. It was possible since for adjusted values of pressure kinematic conditions imposed on a frame were not violated. Sometimes the necessity of keeping deformation of the structure in allowable range causes that objective function can not be obtained with such precision.

5 USING THE LOWEST VALUES OF PRESSURE

The subsequent problem considered is adapting the structure to assumed impact (of energy not exceeding those found in Sec. 3) by using smallest possible inflation of the structure or the smallest amount of introduced energy. The pressures will be adjusted under assumption that they are constant during the whole process, but then their release will be realized as planned in Sec. 4.

Let us initially define space of our optimization problem. By analogy to (5) we can define function $S_p(m)$ returning the subset of Ω_p which contains pressure vectors causing admissible deformation after impact of the given mass:

$$S_p(m) \subseteq \Omega_p \qquad (20)$$

$$S_p(m) = \{\mathbf{p} \mid \mathbf{q}(m,\mathbf{p},t) \in S_q\}$$

For the small value of mass set $S_p(m)$ may overlap with whole set Ω_p. For larger mass set of admissible pressures becomes smaller and may not contain point $\mathbf{p} = 0$. In such situation pressurization of the structure is necessary to sustain given impact. Finally critical mass exists for which only one combination of pressures is possible (cf. Sec. 3). For mass higher than $m_{\max}$ there is no solution of the problem considered. Pressure necessary to avoid beams collision is higher than pressure causing maximal deflection of the lower span.

This time the objective function is expressed explicitly by design variables and it is defined as a sum of pressures or a sum of their squares. In the case of decreasing pressure the objective function is expressed in the form of an integral, cf. (21). Optimization problem can be defined in the following way:

$$\text{Find: } \min\{\Phi(\mathbf{p})) \mid \mathbf{p} \in S_p(m)\} \qquad (21)$$

$$\Phi(\mathbf{p}(t)) = \sum p_i V_i \text{ or } \Phi(\mathbf{p}(t)) = \sum (p_i)^2 V_i \text{ or}$$

$$\Phi(\mathbf{p}(t)) = \int_0^{t_{end}} \sum p_i(t) V_i dt \text{ or } \Phi(\mathbf{p}(t)) = \int_0^{t_{end}} \sum p_i(t)^2 V_i dt$$

where i=1...k and k is the number of cells in APS. The constraints imposed on admissible values of pressures defined by function $S_p(m)$ remain nonlinear. Typical methods for such cases are based on searching of the admissible domain limit because maximum of the objective function is expected to be found there. Such methods are for instance moving along the boundary and tracking active constraints, cf.[7]

By analogy to (10) we can define functions of mass $\Gamma^1_{Sp}(m)$ which returns subsets of $S_p(m)$ containing pressures referring to deformations Γ^1_{Sq}

$$\Gamma^1_{Sp}(m) \subseteq S_p(m) \qquad (22)$$

$$\Gamma^1_{Sp}(m) = \{\mathbf{p} \mid \mathbf{q}(m,\mathbf{p},t) \in \Gamma^1_{Sq}\}$$

and scalar function of pressure $\Gamma_{Sm}^{2}(m)$ which returns subset of $S_p(m)$ containing pressures referring to deformations Γ_{Sq}^{2}

$$\Gamma_{Sp}^{2}(m) \subseteq S_p(m) \tag{23}$$
$$\Gamma_{Sp}^{2}(m) = \{\mathbf{p} \mid \mathbf{q}(m,\mathbf{p},t) \in \Gamma_{Sq}^{2}\}$$

From the shape of plots in Fig. 2 we can deduct that boundary of set $S_p(m)$ will consist of subsets $\Gamma_{Sp}^{1}(m)$, $\Gamma_{Sm}^{2}(m)$ indicating limit deformations and lines $\Gamma_p^{\min}$, $\Gamma_p^{\max}$ which arbitrarily restricts set Ω_p. In general case, boundary of set $S_p(m)$ is not smooth. Minimum of the objective function is expected to be achieved on the line (within set) $\Gamma_{Sp}^{1}(m)$ since this line is located closest to the beginning of the pressures plane.

We assume that the function $\Gamma_{Sm}^{1}(\mathbf{p})$ defined by (10) can be aproximated by 'linear' equation in the form:

$$\Gamma_{Sm}^{1}(\mathbf{p}) = a_0 + \sum_{i=1}^{k} a_i p_i \tag{24}$$

where a_0 is the mass which can be applied to the structure with no pressurization, a_i indicates influence of the pressure p_i on the value of the maximal mass $\Gamma_{Sm}^{1}(\mathbf{p})$. Surface $\Gamma_{Sm}^{1}(\mathbf{p})$ can be approximated basing on only several known values of mass. To obtain better precision it can be also approximated by function of higher order. We have to calculate intersection of this surface with the surface of constant mass m^* finding this way line $\Gamma_{Sp}^{1}(m)$:

$$a_0 + a_1 p_1 + a_2 p_2 = m^* \Rightarrow p_2(p_1) = -\tfrac{a_1}{a_2} p_1 + \tfrac{m^* - a_0}{a_2} \tag{25}$$

The solution must be placed within admissible domain of pressures $S_p(m)$. If the mass m^* is close to $m_{\max}$ found in Sec. 3 we have to search through the line (25) starting from point $p_1 = 0$ and check for each combination of pressures whether the condition Γ_{Sq}^{2} is not violated. Such situation may occur when line $\Gamma_{Sp}^{1}(m)$ indicating lower limit of admissible domain intersects line Ω_p^{12} where both limit conditions imposed on deformation are fulfilled. When this lines intersect twice or more we are dealing with optimization over domain which is not continuous.

Further we will analyze three cell APS loaded in the middle of the upper span previously examined in Sec. 3. For the surface depicted in Fig 2a coefficients a found by the least squares method are equal: $a_0 = 7596\,\text{kg}$, $a_1 = 4{,}736\,\tfrac{kg \cdot m}{kN}$, $a_2 = 27{,}009\,\tfrac{kg \cdot m}{kN}$. In this example we will assume that $m^* = 40000\,kg$ which is lower than any other mass situated on line Ω_p^{12} so we will search minimum sum of pressures applied along the line defined by (25) in the range of $\mathbf{p} \in \Omega_p$. We obtain a problem of linear programming with objective function given by (26):

$$\Phi(\mathbf{p}) = 2 p_1 V_1 + p_2 V_2 = (2V_1 - \tfrac{a_1}{a_2} V_2) p_1 + \tfrac{m^* - a_0}{a_2} V_2 \tag{26}$$

In this situation $a_1 << a_2$ so the objective function is increasing in terms of p_1 and its minimum is achieved for $p_1 = 0$. Value of p_2 obtained from (25) is equal to $1199{,}7\,\frac{kN}{m}$. From formulation (26) we conclude that packages dimensions have significant influence on the results. It is planed to introduce packages dimensions as a design variable in future considerations.

The same problem was also solved for objective function defined in quadratic form:

$$\Phi(\mathbf{p}) = 2(p_1)^2 + (p_2)^2 = \left[2 + (\tfrac{a_1}{a_2})^2\right](p_1)^2 - \left[2 \tfrac{a_1}{a_2} \tfrac{m^* - a_0}{a_2}\right] p_1 + (\tfrac{m^* - a_0}{a_2})^2 \tag{27}$$

where volumes of the packages were neglected since they are equal.. Function $\Phi(\mathbf{p})$ achieves its minimum for $p_1 = 103{,}1\,\frac{kN}{m}$. Value of p_2 obtained from (25) is equal to $1181{,}7\,\frac{kN}{m}$. Hence, the formulation of the objective function as a sum of pressure squares results in distributing small amount of pressure to lateral cells in optimal design.

The problem was solved also for the frame with elastic partitions. Surface $\Gamma_{Sm}^1(\mathbf{p})$ was in this case more sensitive to the value of pressure p_1 and it was possible to absorb wide range of impact by using only this pressure. However, obtained results indicate that pressurizing middle package is the most beneficial.

6 ASSUMED DEFORMATION OF THE STRUCTURE

In the last issue, the considered objective function is based on final deformation. Our goal is to get all packages crushed i.e. obtain minimal distance between upper and lower beam in every cell (cf. Fig 5). In such situation the largest quantity of the gas has to flow out from the structure and the largest amount of energy is dissipated. We control kinematics of the structure as in Sec. 4, however, the objective function (assumed deformation) is not defined in every moment in time. The second difference is that the deformation of the whole structure is assumed, not only displacements of one node.

Mathematical formulation of the problem is given :

$$Find\ \min\left\{\Phi(\mathbf{p}(t)) \middle| \mathbf{p}(t) \in \Omega_p\right\} \tag{28}$$

$$where:\ \Phi(\mathbf{p}(t)) = \sum_{i=1}^{n} D_i(q_u(t_x), q_l(t_x))$$

The objective function is defined as a distance between two nodes belonging to opposite spans which are closest to each other. We try to diminish this distance to zero. Unfortunately, we do not know which nodes will collide and when will it happen (this time could be different for each package). We also cannot decompose the problem to the series of simpler ones. For the simplicity we assume linear decrease of pressure so we have only two design variables in every package. In this formulation we do not assume admissible shape of deformation based on condition of collision between nodes Γ^1_{Sq} and maximal displacement Γ^2_{Sq}. Deformation of the structure does not have to be contained in the set S_q defined by (3).

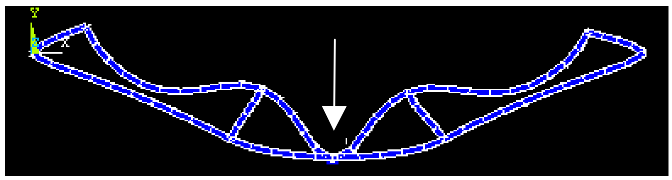

Figure 5: Desired deformation of the structure obtained by means of negative pressures

The best results for such derived optimization problem were obtained for three cell APS with stiff partitions and one sliding support, cf. Fig. 6. By means of compressed air we were able to control places of arising plastic hinges. The structure becomes sensitive to the value of pressure and deformation shape changes. In the pictures we observe difference between fast and slow release of pressure in the middle cell. Applying this release of pressure we are able to distribute central concentrated loading into two sides of the frame. Limitation of pressure to positive values causes that crushing all three packages is not feasible.

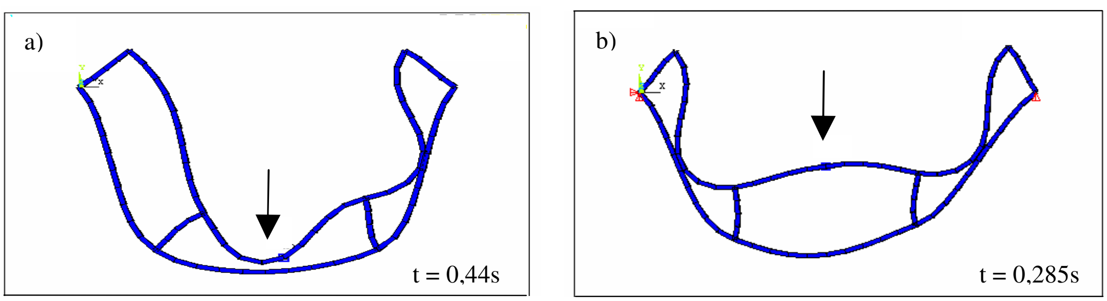

Figure 6: Deformation corresponding to fast (a) and slow (b) release of pressure in the middle cell.

Finally, the problem of optimal pressure distribution for different position of loading was examined. The strategy of filling with pressure is strongly dependent on the scheme of impact so load identification is very important at the preliminary stage of the whole process. In this case our goal was to distribute the destruction caused by lateral impact into two adjacent packages. In both cases external packages are most exposed to destruction so we pressurize it strongly at the initial stage of impact. Then we can release pressure, but we have to move it to the middle cell to avoid its destruction, cf. Fig. 7.

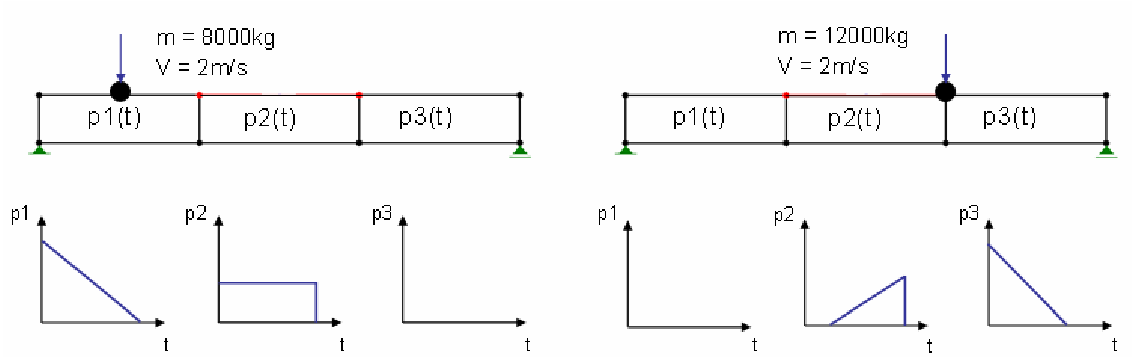

Figure 7: Strategy of the pressure distribution during lateral impact: a) over left chamber, b) over right partition

7 CONCLUSIONS

- APS is based on a concept of pressurized packages within thin-walled structure and controlled pressure outlets (cf. patent pending [8]).
- Pressurized structure can be easily adapted to different load conditions by means of appropriately chosen change of pressure in every package.
- Dynamic properties of APS can be formulated as nonlinear optimization problem
- Optimization algorithms were used to:
  a) increase energy absorption properties
  b) alleviate results of impact
  c) introduce the smallest energy to the system
  d) control kinematic response of the structure

APS equipped with sensors able to detect and identify (in real time) the impact load can serve as on-line adaptive impact absorbing system controlling injection and release of pressure in structural sections. The optimal control strategies have to be pre-computed and stored in memory of hardware controllers to make the process feasible. Motivations for the particular optimization problems discussed in the paper are the following.

a) Maximization of load capacity (chapter 3) is useful in the process of designing of the APS system serving for a given range of loads
b) and c) Smoothing impact absorption (chapter 4) or minimizing the gas pressure (chapter 5) can be used as the real time strategy of adaptation to the detected (and identified) impact load
c) The formulation presented in chapter 6 can be treated as an alternative for the problem discussed in chapter 3.

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